Q1

To find the intersection of two graphs we solve their equations simultaneously.

The second equation is quadratic so rearrange the first (linear) equation into $x =$ or $y =$ form and substitute.

$$x = y + 3$$

[1]

Substitute this into the second (quadratic) equation.

$$3(y+3)^2 - y^2 + (y+3)y = 9$$

[1]

Expand and simplify.

$$3(y^2 + 6y + 9) - y^2 + y^2 + 3y - 9 = 0$$
$$3y^2 + 21y + 18 = 0$$

[1]

If you spot it, we can divide this equation by 3 to simplify the values involved.

$$y^2 + 7y + 6 = 0$$

Solve this by factorising, or if you can't spot the factors, use the quadratic formula (or your calculator).

$$(y+6)(y+1) = 0$$
$$y = -6, \quad y = -1$$

[1]

Substitute these y values into any of the original equations (the rearranged linear equation is usually easiest) to find the x values.

$$x = -6 + 3 = -3 \text{ and } x = -1 + 3 = 2$$

[1]

Be careful to pair up the final answers correctly but the pairs of answers form the coordinates of points P and Q.
(It doesn't matter which round P and Q are.)

$$P\,(-3, -6)$$
$$Q\,(2, -1)$$

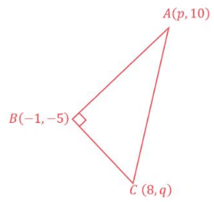
The midpoint of a line between two points is given by taking the (mean) averages of the coordinates.

$$\text{Midpoint of } PQ \text{ is } \left(\frac{-3+2}{2}, \frac{-6+-1}{2}\right) = \left(-\frac{1}{2}, -\frac{7}{2}\right) = (-0.5, -3.5)$$

The coordinates of the midpoint of PQ are $(-0.5, -3.5)$ [1]

Q2

Sketching a quick diagram may help to see the connection between the coordinates and to make sure you are using the correct gradients in the correct places.



The gradient of AC can be found in terms of p and q and set equal to $-\frac{6}{7}$.

$$m_{AC} = \frac{10 - q}{p - 8} = -\frac{6}{7}$$

Expression for the gradient [1]
Forming the equation [1]

Angle ABC is a right angle, so AB and BC are perpendicular to each other.

The gradients of perpendicular lines are negative reciprocals so the gradients of AB and BC can be found in terms of p and q.

$$m_{AB} = \frac{10 - -5}{p - -1} = \frac{15}{p+1}$$
$$m_{BC} = \frac{q - -5}{8 - -1} = \frac{q+5}{9}$$

[1]

Set the gradient of AB equal to the negative reciprocal of the gradient of BC.

Set the gradient of AB equal to the negative reciprocal of the gradient of BC.

$$m_{AB} = \frac{1}{m_{BC}}$$
$$\frac{15}{p+1} = -\frac{1}{\frac{q+5}{9}} = -\frac{9}{q+5}$$

[1]

Solve the two equations simultaneously to find the values of p and q.

Number the equations.

$$\frac{10 - q}{p - 8} = -\frac{6}{7} \qquad (1)$$
$$\frac{15}{p+1} = -\frac{9}{q+5} \qquad (2)$$

Rearrange equation (1) to get rid of the fractions.

$$7(10 - q) = -6(p - 8) \qquad (1)$$

Expand the brackets on both sides of equation (1).

$$70 - 7q = -6p + 48 \qquad (1)$$

Simplify.

$$6p - 7q = -22 \qquad (1)$$

Rearrange equation (2) to get rid of the fractions.

$$15(q+5) = -9(p+1) \qquad (2)$$

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$$15(q+5) = -9(p+1) \qquad (2)$$

Expand the brackets on both sides of equation (1).

$$15q + 75 = -9p - 9 \qquad (2)$$

Simplify by bringing unknown terms to one side.

$$9p + 15q = -84 \qquad (2)$$

Multiply equation (1) by 3 and equation (2) by 2 and eliminate *p*.

$$\begin{array}{rrcll} & 18p - 21q & = & -66 & 3(1) \\ (-) & 18p + 30q & = & -168 & 2(2) \\ \hline & -51q & = & 102 & \end{array}$$

[1]

Solve to find *q*.

$$q = \frac{102}{-51} = -2$$

Find *p* by substituting $q = -2$ into equation (1) and solving.

$$\begin{aligned} (1) \qquad 6p - 7(-2) &= -22 \\ 6p + 14 &= -22 \\ 6p &= -36 \\ p &= -6 \end{aligned}$$

$\mathbf{p = -6, q = -2}$ [1]

Q3

Use the information $JK = \sqrt{80}$ to find an equation for the length of the line connecting *J* and *K* in terms of *j* and *k*.

The formula for the length between the points (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. Use this formula with the points (*j*, 15) and (6, *k*).

$$\begin{aligned} \sqrt{(k-15)^2 + (6-j)^2} &= \sqrt{80} \\ (k-15)^2 + (6-j)^2 &= 80 \end{aligned}$$

[1]

The line connecting the top vertex of an isosceles triangle (*H*) to the midpoint of its base (*JK*) is perpendicular to the base, so the lines *HM* and *JK* are perpendicular.

The gradients of perpendicular lines are negative reciprocals of each other.

$$m_{JK} = -\frac{1}{m_{HM}} = -\frac{1}{2}$$

[1]

The gradient of *JK* can be found in terms of *j* and *k* and set equal to $-\frac{1}{2}$.

$$m_{JK} = \frac{15-k}{j-6} = -\frac{1}{2}$$

[1]

Solve the two equations simultaneously to find the values of *j* and *k*.

Number the equations.

$$\begin{aligned} (k-15)^2 + (6-j)^2 &= 80 \qquad (1) \\ \frac{15-k}{j-6} &= -\frac{1}{2} \qquad (2) \end{aligned}$$

Notice here that the expressions inside the brackets in equation (1) are similar to those on the numerator and denominator of equation (2). Multiply both parts of the fraction by -1 to get the expressions to be the same.

Notice here that the expressions inside the brackets in equation (1) are similar to those on the numerator and denominator of equation (2). Multiply both parts of the fraction by -1 to get the expressions to be the same.

$$\frac{-(15-k)}{-(j-6)} = -\frac{1}{2} \quad (2)$$

$$\frac{k-15}{6-j} = -\frac{1}{2} \quad (2)$$

Rearrange equation (2) to make (k -15) the subject.

$$k - 15 = -\frac{1}{2}(6-j) \quad (2)$$

Substitute into equation (1).

$$\left(-\frac{1}{2}(6-j)\right)^2 + (6-j)^2 = 80 \quad (1)$$

$$\left(\frac{j}{2} - \frac{6}{2}\right)^2 + (6-j)^2 = 80 \quad (1)$$

$$\left(\frac{j}{2} - 3\right)^2 + (6-j)^2 = 80 \quad (1)$$

Expand the brackets and simplify.

$$\left(\frac{j}{2}\right)^2 - 6\left(\frac{j}{2}\right) + 9 + 36 - 12j + j^2 = 80 \quad (1)$$

$$\frac{j^2}{4} - 3j + 45 - 12j + j^2 = 80 \quad (1)$$

$$\frac{5j^2}{4} - 15j + 45 = 80 \quad (1)$$

Rearrange equation (1) to form a quadratic in the form $ax^2 + bx + c = 0$

$$5j^2 - 60j + 180 = 320 \quad (1)$$

$$5j^2 - 60j - 140 = 0 \quad (1)$$

[1]

Equation (1) can be made easier to solve by dividing every term by 5.

$$j^2 - 12j - 28 = 0 \quad (1)$$

Factorise.

$$(j-14)(j+2) = 0 \quad (1)$$

$$j = 14 \text{ or } j = -2$$

[1]

We are told that $j < 0$, so ignore the positive answer and use the negative value to find the corresponding value of k by substituting into equation (2).

$$k - 15 = -\frac{1}{2}(6-j) \quad (2)$$

$$k - 15 = -\frac{1}{2}(6-(-2)) \quad (2)$$

$$k = -\frac{1}{2}(8) + 15 \quad (2)$$

$$k = 11$$

$\mathbf{j = -2, k = 11}$ [1]

Q4

Find the coordinates of the points *A* and *B* by solving the two equations simultaneously.

Only one of the equations is linear, so rearrange this into $x=$ or $y=$ form and substitute it into the second equation.

$$x = y - 4$$

Substitute this into the second equation to form a quadratic.

$$(y - 4)^2 - (y - 4) + y^2 = 10$$

[1]

Expand and simplify.

$$y^2 - 4y - 4y + 16 - y + 4 + y^2 = 10$$
$$2y^2 - 9y + 20 = 10$$

Rearrange to form a quadratic equation that can be solved.

$$2y^2 - 9y + 10 = 0$$

[1]

Solve this by factorising, or if you can't spot the factors, use the quadratic formula.

$$(2y - 5)(y - 2) = 0$$
$$y = \frac{5}{2}, \quad y = 2$$

[1]

Substitute these *y* values into any of the original equations (the rearranged linear equation is usually easiest) to find the *x* values.

$$x = \frac{5}{2} - 4 = 2.5 - 4 = -1.5 \text{ and } x = 2 - 4 = -2$$

Be careful to pair up the final answers correctly but the pairs of answers form the coordinates of points *A* and *B*.
(It doesn't matter which round *A* and *B* are.)

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(It doesn't matter which round *A* and *B* are.)

$$A\ (-1.5, 2.5)$$
$$B\ (-2, 2)$$

[1]

The length of a line segment between two points is found using Pythagoras' theorem, substitute the coordinates for *A* and *B* into the formula $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

$$\sqrt{(-2 - (-1.5))^2 + (2 - 2.5)^2}$$

[1]

You could simplify before typing it into your calculator or just type it straight in, remember that your answer must be given in surd form.

$$\sqrt{(0.5)^2 + (-0.5)^2}$$
$$= \sqrt{0.25 + 0.25}$$
$$= \sqrt{0.5}$$
$$= \sqrt{\frac{1}{2}}$$
$$= \frac{1}{\sqrt{2}}$$

Give your final answer in the form given in the question, most calculators will give this form automatically when you type in the full calculation, if not you will need to rationalise the denominator.

$$\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$\frac{\sqrt{2}}{2}$ [1]

Q5

To find the intersection of two graphs we solve their equations simultaneously.

The second equation is quadratic so rearrange the first (linear) equation into $x =$ or $y =$ form and substitute. For this equation it is easiest to make y the subject.

$$y = 6x$$

As both equations are equal to y, eliminate y by setting the two equations equal to each other.

$$(10x - 3)(x + 1) = 6x$$

[1]

Expand the double brackets on the left-hand side and simplify.

$$10x^2 + 10x - 3x - 3 = 6x$$
$$10x^2 + 7x - 3 = 6x$$

Rearrange to form a quadratic by subtracting $6x$ from both sides.

$$10x^2 + 7x - 6x - 3 = 0$$
$$10x^2 + x - 3 = 0$$

[1]

Solve this by factorising if you can, or use the quadratic formula, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where $ax^2 + bx + c = 0$.

Be sure to **show** the stages of using the quadratic formula - only use your calculator to check your answers.

$$a = 10,\ b = 1,\ c = -3$$

$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4 \times 10 \times (-3)}}{2 \times 10}$$

[1]

$$x = \frac{-1 \pm \sqrt{121}}{20} = \frac{-1 \pm 11}{20}$$
$$x = \frac{-12}{20} = -\frac{3}{5} = -0.6$$

and

$$x = \frac{10}{20} = \frac{1}{2} = 0.5$$

[1]

Substitute these x values into any of the original equations (the rearranged linear equation is usually easiest) to find the y values.

$$y = 6x = 6\left(-\frac{3}{5}\right) = -\frac{18}{5} = -3.6$$

and

$$y = 6x = 6\left(\frac{1}{2}\right) = 3$$

Be careful to pair up the final answers correctly but the pairs of answers form the coordinates of points A and B.
(It doesn't matter which round A and B are.)

$$A\ (-0.6,\ -3.6)$$
$$B\ (0.5,\ 3)$$

The midpoint of a line between two points is given by taking the (mean) averages of the coordinates.

Midpoint of AB is $\left(\dfrac{-0.6 + 0.5}{2}, \dfrac{-3.6 + 3}{2}\right) = \left(-\dfrac{0.1}{2}, -\dfrac{-0.6}{2}\right) = \left(-\dfrac{1}{20}, \dfrac{-3}{10}\right)$

[1]

The coordinates of the midpoint of AB are $(-0.05,\ -0.3)$ [1]